SYNOPSIS

Digital topology plays a very important role in computer vision, image processing and computer graphics. It can be defined as the application of topological ideas to image arrays. Digital pictures are rectangular arrays of non negative numbers. Generally digital topology deals with the question of how and to what extend topological concepts can meaningfully and usefully be applied to a binary image.

The subject digital topology has been studied since the late 1960's when it was introduced by Rosenfeld. Initially all work in digital topology was based on graph theoretic rather than topological notion. At the end of 80's, V. Kovalevsky gave a sound reason for digital topology which again turned out to be a part of Alexandroff theory. The third approach to digital topology was suggested by Khalimsky in 1980's. Khalimsky's topology on Z^n is the product of the n-copies of topology on Z for which $\{2i-1, 2i, 2i+1/i \in Z\}$
is a sub-base. A drawback of the Khalimsky topology is that the Jordan curves with respect to it can never turn at an acute angle. To overcome this deficiency another topology was introduced by Slapal.

The digital fundamental group of discrete object was introduced in Digital topology by Kong. Boxer shows how classical methods of Algebraic topology may be used to construct the digital fundamental group. Knowledge of the digital fundamental group is a very important tool for image analysis.

In the present study we discuss some topological properties (connectedness, continuity etc.) of digital images with respect to Khalimsky topology and its generalizations. We also study some properties of digital homotopy and digital fundamental group.

This thesis is organized in six chapters. The first chapter contains some preliminary concepts, which are required for the subsequent chapters.

Though most of the materials that are presented here are well-known, they are included only for completeness and quick reference.

In chapter 2, Marcus topology, Khalimsky topology and Slapal's topologies are discussed. We also prove that Khalimsky topology and Marcus topology are quotient topologies of Slapal's topology and Marcus topology can be obtained from Khalimsky topology by using a transformation.

In Chapter 3, connectedness and continuity in topological spaces with respect to Khalimsky topology are discussed. Properties of continuous functions from $Z \to Z$, $Z^n \to Z$ are also discussed. Like in Real Analysis, Intermediate value theorem and fixed point theorem for the Khalimsky line are discussed. Some characterizations of smallest neighbourhood space are also discussed. Properties of locally finite spaces are discussed in section 3.5. Properties of extended Khalimsky line is studied in section 4.7.

Chapter 4 deals with several continuous functions from $Z^n \to Z^n$. A Khalimsky continuous map $f: X_{n_0,k_0} \to Y_{n_1,k_1}$ need not preserve the k_0-connectivity of X into k_1-connectivity of Y, where $n_0 \geq 2$, $n_1 \geq 1$. So we extend the definition by finding another continuity which preserves the k_0-connectivity into k_1-connectivity. Digital version of the pasting theorem is also established.

In chapter 5, we study various properties of digital homotopy and digital contraction. We have proved that digital homotopy is an equivalence relation among digitally continuous functions.

In chapter 6, we discuss properties of simple closed curves having at least four points. In section 6.2, properties of digital fundamental groups are discussed. In section 6.3, we proved that a pair of digital simple closed curves need not have the same digital homotopy type.

Finally, the last chapter contains the conclusion of the work and scope of future investigation. We do not claim that our investigation study is complete and exhaustive in every respect. There are so many related areas to be investigated. We hope this will create an interest in further investigation.

Since it is an application field the future of digital topology lies in analysis of the properties like connectedness, adjacency, digital homotopy etc. with different fields like pattern recognition, medical imaging, image processing, neuroscience, geoscience etc.